Scenario Based Robust-Optimization of Tree Structures

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I. Robust Binary Search Trees

Problem Formulation

Given

- n keys
- k scenarios $F^1, \ldots, F^k \in \mathbb{R}_+^n$ (a.k.a. frequency vectors)

Objective

To construct a single binary search tree (BST) of best-possible performance, relative to any scenario.

Possible perfomance metrics:

- $\min_T \max_{s\in\{1,\ldots,k\}} \text{cost}(T, F^s)$ *(worst-case cost);*
- $\min_T \max_{s\in\{1,\ldots,k\}} \left\{\text{cost}(T, F^s)/\text{OPT}(F^s)\right\}$ (*competitive ratio*);
- $\min_T \max_{s\in\{1,\ldots,k\}} \left\{\text{cost}(T, F^s) - \text{OPT}(F^s)\right\}$ (*regret*),

where

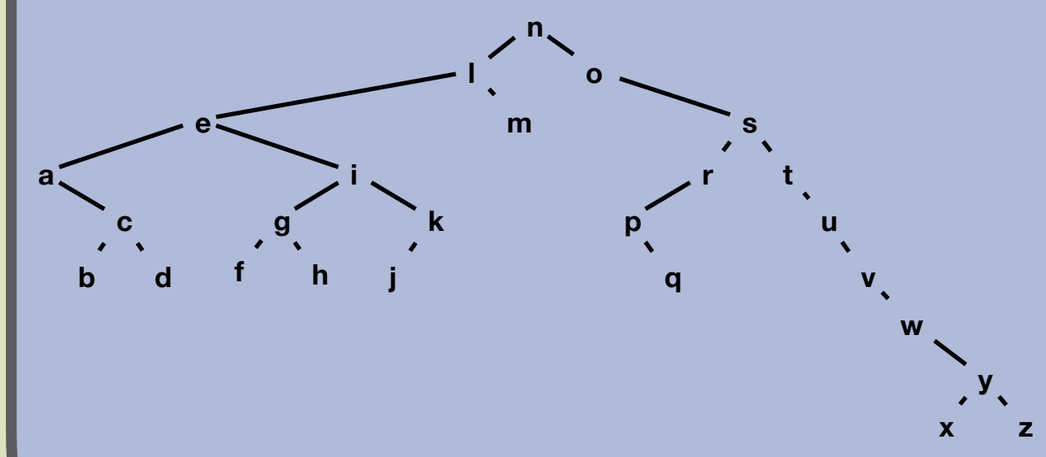
$$\text{cost}(T, F^s) := \sum_{i=1}^{n} F_i^s \cdot [\text{level of } i \text{ in } T]\,.$$

Example

Given

- 26 keys: {a, …, z}
- 10 scenarios 🇩🇰 🇳🇱 🏴 🇫🇮 🇫🇷 🇩🇪 🇮🇹 🇵🇹 🇪🇸 🇸🇪

Optimal tree for competitive ratio:



Results

- This problem is **NP-hard**, even if $k = 2$ (for all 3 metrics).
- We obtain an **optimal** $\lceil \log_2(k+1) \rceil -$ approximation **algorithm** for competitive ratio.
- We provide **mixed integer linear program (MILP)** formulations to compute optimal trees with *Gurobi* and compare them with our algorithms.
- Similar results for **Robust Huffman Trees** are achieved.

II. Regret and Fairness

Simpler problem

Given

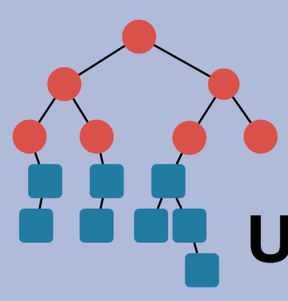
- n keys of two types 🔴/🟦 (e.g., 👧/👨, 🇫🇷/🇪🇸)
- 2 scenarios F^1: uniform on 🔴, F^2: uniform on 🟦.

regret$(T, F^1) := \text{cost}(T, F^1) - \text{OPT}(F^1)$ (*red regret*)
regret$(T, F^2) := \text{cost}(T, F^2) - \text{OPT}(F^2)$ (*blue regret*)

Objective

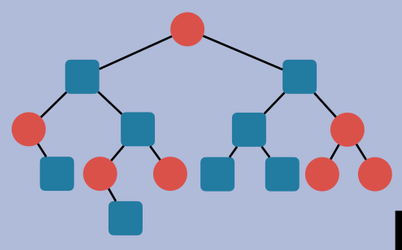
To construct a single BST such that red regret and blue regret are bounded. In other words, both types of keys are treated in a **fair manner.**

For 🔴🟦🟦🔴🟦🟦🔴🔴🟦🟦🟦🔴🔴🔴, two of the possible trees:



Unfair tree

Red regret = 0
Blue regret = 17

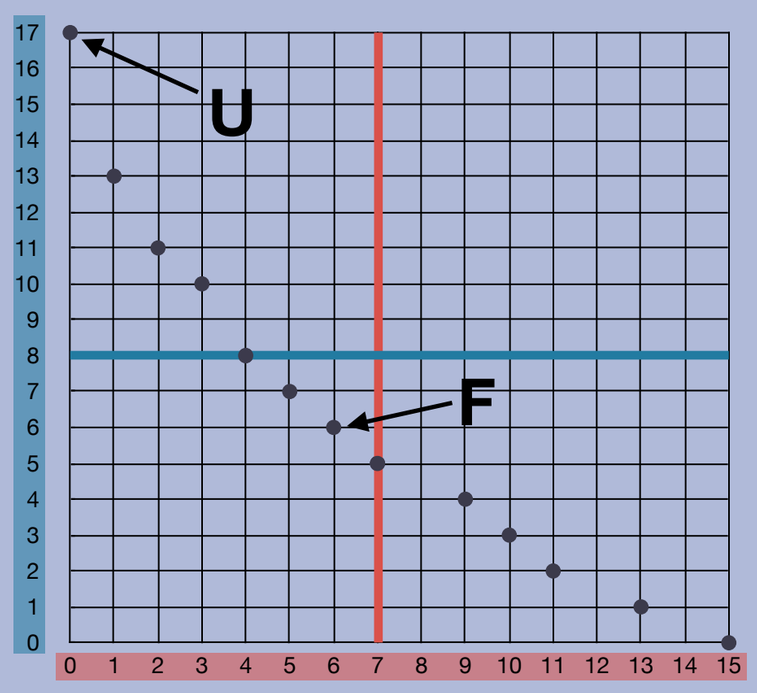


Fair tree

Red regret = 6
Blue regret = 6

Results

We provide an algorithm for computing the pareto frontier for k scenarios with uniform distribution.



Conjecture

There is always a tree T such that

$$\text{red cost in } T \leq \text{OPT}(\#\text{red}) + 1$$

$$\text{blue cost in } T \leq \text{OPT}(\#\text{blue}) + 1$$

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